On the theory of gravitation field

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Abstract

We construct a general relativity formula for the law of gravity for material bodies. The formula contains three numeric parameters that are to be determined experimentally. If they are chosen from symmetry considerations, then the theory that appears is close to the theory of electrodynamics: the gravitational field is given by two vector fields, one can write the energy-momentum tensor, we give an answer on the question what a gravitational wave is. Going to infinity, this wave carries with it the negative energy.

The author's platform

The author divests himself of the official ideology on the subject [1].

The vacuum is a physical medium. It can be acted upon, and its condition can be changed. The gravitational field is a special state, special deformation of the vacuum, and this field is not the main, most important physical field. The actual energy, the mass as a synonym for the energy, does not generate a gravitational field. But it can be generated by physical bodies, since they have at their disposal the corresponding mechanism of interaction with the vacuum. Naturally, elementary particles (not all) should also have this mechanism. These particles, from their "birth", sit on the vacuum and are accompanied by gravitational fields. The energy by itself has nothing of the sort. The electron has a mass, but this does not mean that it possesses its own gravitational field. The author is afraid that the mystery of the strange

constant $2.4 \cdot 10^{-43}$ could turn out to be a miscomprehension. The same can be said about photons, with the remark that there is no experimental data showing that these particles possess a gravitational field. The author would consider it a great miracle, if photons passed by the Sun without a trajectory curving. There could be a dozen of reasons to prohibit photons to move in a straight line.

The author claims that the gravitational field of a physical body is determined, first of all, by the quantity and quality of elementary particles that make the body. This is precisely at this point where the notion of a heavy mass appears as an integral characteristic of the body. A commonly accepted in physics hypothesis is that a heavy and inertial masses are equivalent. This hypothesis does not contradict to known at this point experimental data. It also looks very attractive. But the author does not believe that the beauty will save the world.

In this article, we accept that the notions of a heavy and inertial mass are unequal. The formulas and conclusions proposed in this article do not either contradict to any known experimental facts.

We take the following well known formulation as a fundamental one.

Let Ω be some bounded region in R^3, and σ_0 be the density of a distributed in Ω heavy mass. Let φ be a solution of the equation $\Delta\varphi = 4\pi\kappa\sigma_0$, and $\varphi \to 0$, $|r| \to \infty$. Then there exists κ such that the formula $\mathbf{g} = -\sigma_0\nabla\varphi$ gives the force density with which the gravitational field acts on a heavy mass in an elementary volume. This parameter, κ, is called gravitational constant, and the function φ is the potential of the gravitational field.

Relativistic formulas

We need to obtain a relativistic formula for the force density $\mathbf{g}$ for a moving heavy mass. At this point we suppose that the motion of the heavy mass can be determined by the action of external, nongravitational forces. In other words, we suppose that the velocity field of the heavy mass, v_α, is a quantity by itself in the theory.

Denote by V_k the 4-velocity[1] of the particles, and by σ_0 — the density of the heavy mass relatively to an IRF in which the particles are stationary.

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[1] Here and in the sequel, $V_\alpha = \gamma v_\alpha$, $V_4 = ic\gamma$, $(x_1, x_2, x_3, x_4) = (x, y, z, ict)$, $\partial_n = \partial/\partial x_n$.

Let us form a 4-vector, $s_k = \sigma_0 V_k$, and call it the 4-flux of the heavy mass. At this point it is almost necessary to replace the equation $\Delta \varphi = 4\pi \kappa \sigma_0$ with the system

$$\partial_n^2 \Phi_k = -4\pi c^{-2} \kappa s_k, \qquad k = 1, 2, 3, 4, \tag{1}$$

The solution of this system subject to $\Phi_k(r, t) \to 0, |r| \to \infty, k = 1, 2, 3, 4,$ will be called 4-potential of the gravitational field induced by the 4-flux of the heavy mass s_k. In the case where the masses are stationary, we get $\partial_k^2 \Phi_4 = \Delta \Phi_4 = -4i\pi c^{-2}\kappa\sigma_0$, $\Phi_\alpha \equiv 0$, whence $\Phi_4 = -ic^{-1}\varphi$, $g_\alpha = -ic\sigma_0\partial_\alpha\Phi_4$. It is naturally to assume that the force we are searching for, $\mathbf{g}$ (and the 4-force g_k) should depend, in the general case, only on first order derivatives $\partial_j \Phi_n$, that it should be proportional the density σ_0, depend in a certain way on V_j and, maybe, on $\partial_n V_j$. To find such a dependence, we use the necessary condition $g_k V_k = 0$, which yeilds $g_k = \Theta_{ks} V_s$ with some antisymmetric matrix Θ_{ks}. It is clear that this matrix is generated by the quantities in the theory and must have a correct dimension. This leads to the following possible ways to construct the matrix Θ_{ks}:

$$\begin{aligned}
\Theta_{ks}(1) &= -\sigma_0(\partial_k\Phi_s - \partial_s\Phi_k), \\
\Theta_{ks}(2) &= \sigma_0(p_{sn}\partial_n\Phi_k - p_{kn}\partial_n\Phi_s), \\
\Theta_{ks}(3) &= \sigma_0(q_{sn}\partial_k\Phi_n - q_{kn}\partial_s\Phi_n), \\
\Theta_{ks}(4) &= \sigma_0(M_{sk} - M_{ks})\partial_n\Phi_n, \\
\Theta_{ks}(5) &= \sigma_0(N_{skmn} - N_{ksmn})\partial_n\Phi_m.
\end{aligned}$$

All quantities p_{sn}, q_{sn}, M_{sk}, N_{skmn} in these formulas are dimensionless. We have no choice but to construct these expressions by using the dimensionless $c^{-1}V_j$. Thus we get $p_{sm} \sim q_{sm} \sim c^{-2}V_s V_m$, $M_{sk} = M_{ks}$, $N_{skmn} = N_{ksmn}$, so that $\Theta_{ks}(4) = \Theta_{ks}(5) = 0$. Thus three first constructions of the matrix Θ_{ks} remain, and, up to a constant factor,

$$\begin{aligned}
\Theta_{ks}(2) &= \sigma_0 c^{-2} V_n(V_s\partial_n\Phi_k - V_k\partial_n\Phi_s), \\
\Theta_{ks}(3) &= \sigma_0 c^{-2} V_n(V_s\partial_k\Phi_n - V_k\partial_s\Phi_n).
\end{aligned}$$

These three found tensors should be supplemented with the tensor $\Theta_{ks}(1^*) = e_{ksnm}\Theta_{nm}(1)$, where e_{ksnm} is the totally antisymmetric unit tensor. There is no sense in introducing similar $\Theta_{ks}(2^*)$ and $\Theta_{ks}(3^*)$, since $\Theta_{ks}(2^*)V_s = 0$ and $\Theta_{ks}(3^*)V_s = 0$. Finally, there are 4 choices for the formula

3

of the 4-density g_k:

$$
\begin{aligned}
g_k(1) &= -\sigma_0 V_s(\partial_k \Phi_s - \partial_s \Phi_k), \\
g_k(2) &= -\sigma_0 V_n \partial_n \Phi_k - \sigma_0 c^{-2} V_n V_s V_k \partial_n \Phi_s, \\
g_k(3) &= -\sigma_0 V_n \partial_k \Phi_n - \sigma_0 c^{-2} V_n V_s V_k \partial_s \Phi_n, \\
g_k(1^*) &= -\sigma_0 e_{ksnm} V_s(\partial_n \Phi_m - \partial_m \Phi_n).
\end{aligned}
\tag{2}
$$

Remark

We could have assumed that the formula for g_k also contains linear expressions of the potentials Φ_n. Then there would be two more possible constructions for Θ_{ks},

$$
\Theta_{ks}(6) = \sigma_0(a_k \Phi_s - a_s \Phi_k), \qquad \Theta_{ks}(7) = \sigma_0(b_{ksn} - b_{skn})\Phi_n.
$$

Here the quantities a_k and b_{ksn} must have the same dimension as $c^{-1}\partial_n V_j$. This leads to the formulas (up to a constant factor):

$$
\begin{aligned}
\Theta_{ks}(6) &= c^{-2}\sigma_0 V_n(\Phi_s \partial_n V_k - \Phi_k \partial_n V_s), \\
\Theta_{ks}(7a) &= c^{-2}\sigma_0 V_n \Phi_n(\partial_k V_s - \partial_s V_k), \\
\Theta_{ks}(7b) &= c^{-2}\sigma_0 \Phi_n(V_k \partial_n V_s - V_s \partial_n V_k), \\
\Theta_{ks}(7c) &= c^{-2}\sigma_0 \Phi_n(V_k \partial_s V_n - V_s \partial_k V_n).
\end{aligned}
$$

One should also add $\Theta_{ks}(6^*) = e_{ksnm}\Theta_{nm}(6)$ to these tensors, since $\Theta_{ks}(6^*)V_s \not\equiv 0$. This collections of tensors gives the following possibilities for formulas for $g_k = \Theta_{ks}V_s$:

$$
\begin{aligned}
g_k(6) &= c^{-2}\sigma_0 \Phi_s V_s V_n \partial_n V_k, & g_k(7b) &= \sigma_0 \Phi_n \partial_n V_k, \\
g_k(7c) &= \sigma_0 \Phi_n(\partial_k V_n + c^{-2}V_k V_s \partial_s V_n), & g_k(6^*) &= \Theta_{ks}(6^*)V_s.
\end{aligned}
$$

We see that all these g_k can not express the force of the gravitational field, since they all vanish for $v_\alpha = 0$. But these formulas could give an additional term in the formulas containing $\partial_j \Phi_n$. We do not discuss this subject any further.

Let us go back to formulas (2). For a heavy stationary mass, we have

$$
g_k(1) = -ic\sigma_0 \partial_k \Phi_4, \qquad g_k(2) = 0, \qquad g_k(3) = g_k(1), \qquad g_k(1^*) = 0.
$$

Thus formulas for $g_k(2)$ and $g_k(1^*)$ could also give additional terms in formulas for $g_k(1)$ and $g_k(3)$, and the final formula for g_k has the form:

$$g_k = \lambda g_k(1) + (1 - \lambda)g_k(3) + \mu g_k(2) + \nu g_k(1^*). \tag{3}$$

The numbers λ, μ, and ν must be determined from experiments. It is feasible that $\lambda = 1$, $\mu = \nu = 0$. In such a case, we get the formula:

$$g_k = -s_m(\partial_k \Phi_m - \partial_m \Phi_k). \tag{4}$$

In the sequel we assume formula (4) to hold.

The fields F and G of the gravitational field.

Formulas (1) and (4) have the form of the main formulas in classical electrodynamics [2]. And by similarity to electrodynamics, introduce the vector fields $\mathbf{F}$ and $\mathbf{G}$ which, together with the 4-potential Φ_k, are characteristics of the gravitational field. Introduce the notations $\sigma = \sigma_0 \gamma$, $\mathbf{s} = (s_1, s_2, s_3)$, $\mathbf{\Phi} = (\Phi_1, \Phi_2, \Phi_3)$, $\varphi = ic\Phi_4$, $\mathbf{F} = \nabla\varphi - \dot{\mathbf{\Phi}}$, $\mathbf{G} = \mathrm{rot}\, \mathbf{\Phi}$, $\mathbf{g} = (g_1, g_2, g_3)$. Then formulas (4) give the following:

$$\mathbf{g} = -\sigma\mathbf{F} - [\mathbf{s}, \mathbf{G}].$$

It is easy to check that the following relations hold:

$$\mathrm{div}\, \mathbf{G} = 0, \qquad \dot{\mathbf{G}} + \mathrm{rot}\, \mathbf{F} = 0, \qquad \mathrm{div}\, \mathbf{F} = 4\pi\kappa\sigma - \partial_k\dot{\Phi}_k,$$
$$\mathrm{rot}\, \mathbf{G} - c^{-2}\dot{\mathbf{F}} = 4\pi\kappa c^{-2}\mathbf{s} + \nabla(\partial_k\Phi_k). \tag{5}$$

Let $\partial_k s_k = 0$. Then the relationship between the fields $\mathbf{G}$, $\mathbf{F}$, and $\mathbf{s}$ will be the same as between the fields $\mathbf{B}$, $\mathbf{E}$, and $\mathbf{j}$ in classical electrodynamics [2]. Here the condition $\partial_k s_k = 0$ corresponds to the electrical charge conservation law in electrodynamics. In our case, the heavy mass is characterized by the quantity and quality of the particles that make up the body, and if a change of its volume induces a change of its energy, the heavy mass of the body does not need to change. Thus, *the condition that $\partial_k s_k = 0$ is absolutely natural for the 4-flux s_k of the heavy mass, and we add it to the general definition of the heavy mass.*

The energy-momentum tensor

The main equation for the energy-momentum tensor τ_{ks} of the gravitational field is the equation $\partial_s \tau_{ks} = g_k$ with the right-hand side given by $g_k = -s_m(\partial_k \Phi_m - \partial_m \Phi_k)$. The author believes that there are no reasons for defining the notion of the energy-momentum tensor of the gravitational field, since the very notion of the 4-density g_k is not defined. At this point one could give a simple answer to this question. The formulas given here repeat main formulas of electrodynamics, and thus to write down the tensor τ_{ks}, we use the Poynting's tensor T_{ks}[2]. Denoting $L_{km} = \partial_m \Phi_k - \partial_k \Phi_m$, we get:

$$\tau_{ks} = \frac{c^2}{4\pi\kappa} L_{km} L_{sm} - \frac{c^2}{16\pi\kappa} L_{nm}^2 \delta_{ks}.$$

Note that this formula can only be written if $\partial_k s_k = 0$, whereas formula (4) was obtained without this condition. The tensor τ_{ks} defines the density of the gravitational energy in the space $W = \tau_{44}$, flux of the gravitational energy $\mathbf{S} = (S_1, S_2, S_3)$, where $S_\alpha = ic\tau_{4\alpha}$, and the density of the momentum $p_\alpha = ic^{-1}\tau_{\alpha4}$. It is not difficult to calculate that

$$W = -\frac{1}{8\pi\kappa}\mathbf{F}^2 - \frac{c^2}{8\pi\kappa}\mathbf{G}^2, \qquad \mathbf{S} = -\frac{c^2}{4\pi\kappa}[\mathbf{F},\mathbf{G}].$$

Note that the quantity W is negative. This means that the creation of the gravitational field leads to taking energy from the vacuum. And, consequently, unperturbed vacuum has a certain inner reserve of energy. This is not something extraordinary, since vacuum is a medium.

The solution of system (5), $\mathbf{F} = (0, ca_1(x - ct), ca_2(x - ct))$, $\mathbf{G} = (0, -a_2(x - ct), a_1(x - ct))$, is an example of a flat gravitational wave. It has $W = \frac{-c^2}{4\pi\kappa}(a_1^2 + a_2^2)$, $\mathbf{S} = (Wc, 0, 0)$. Going to infinity, the wave takes with it the negative energy.

Let, for example, an electric charge oscillate along certain axis. Then there will be electromagnetic radiation that carries with it positive energy. The source of this energy is in the cause that keeps the process oscillating. If a heavy mass will oscillate about an axis, then, instead of electromagnetic, there will be gravitational radiation. But the system now looses negative energy, i.e. its total energy increases, but, of course, not every kind of the energy increases. And it could be that precisely mechanical oscillations recuperate those material systems that are not able to radiate electromagnetic

waves any more. Let us also note that experiments on detecting gravitational waves will hardly be successful if they are oriented with an expectation that they should carry positive energy. Recall that we set $\lambda = 1$, $\mu = \nu = 0$ in formula (3). For another choice of λ, μ, ν, the theory experiences some difficulties. The formula for $g_k(2)$ contains the third power of velocity, and thus the tensor $\tau_{ks}(2)$ can not be written in terms of σ_0, Φ_n, V_n and their derivatives with the condition that $\partial_s \tau_{ks}(2) = g_k(2)$. It is also impossible to write the tensor $\tau_{ks}(1^*)$ subject to $\partial_s \tau_{ks}(1^*) = g_k(1^*)$ because of a poor geometry of the formula for $g_k(1^*)$.

References

[1] L.D.Landau and E.M.Lifshits, The Classical Theory of Fields, Pergamon Press, Oxford, 1971.

[2] A.J.W.Sommerfeld, Electrodynamics, Academic Press, 1952.